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# ABOUT THE INFLUENCE OF THE INITIAL PHASE OF THE HARMONIC COMPONENTS ON THE CURRENT AMPLITUDE-FREQUENCY SPECTRUM

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**ABSTRACT**

This article is aimed at assessing the influence of the initial phases of sinusoidal components contained in the studied signals on generated amplitude-frequency spectrum. The value of the initial phases of the sinusoidal components is initially unknown. This hamper studying their influence on spectrum formation and, accordingly, on assessing the harmonic components amplitude values with specified accuracy. This article is aimed at obtaining a formula that reflects, with specified initial limitations, formation of the current amplitude-frequency spectrum in time, and considers the influence of the initial phases of sinusoidal components on spectrum formation. According to the obtained formula, the above-mentioned effect takes place and decreases in time with formation of the amplitude-frequency spectrum. Decreasing influence allows to reduce the effect of initial phase on the generated spectrum to the required value. This is achieved by selecting the appropriate values of time intervals for spectra formation, and, in turn, allows to obtain spectra that do not significantly depend on the values of initial phases. Thus, the obtained results may be the basis for obtaining values of spectra and, correspondingly, amplitudes of the sinusoidal components with specified accuracy; they also create the theoretical basis that may be useful for subsequent analytical calculations.

## INTRODUCTION

Spectral analysis is widely used in many fields of human activity. It is used in speech recognition, in noise analysis, in diagnosing technical and biological objects, and in surface and underwater navigation **(**Britenkov and Stepanov, 2014; Jurovich, *et al*., 2014; Muravyev, 2009; Rastegaev, *et al*., 2012; Rybochkin and Yakovlev, 2011**)**. Due to the wide use, the study of spectral analysis properties is paid considerable attention to (Antipov, *et al*., 2014; Diaconitsa, *et al*., 2013). (Lyubomudrov, 2011; Lyubomudrov and Bashkov, 2011; Ponomarev, *et al*., 2014; Filatov and Filatov, 2010).

The basis of spectral analysis in solving the above tasks is formation of the current amplitude-frequency spectrum, which is formed in successive intervals of time [0, *t*]. This spectrum characterizes the amplitude of harmonic components A sin(ωt + φ) , from which analyzed signal f(t) is formed in time interval [0, *t*], and is calculated according to formula:

$$\Phi(t,\omega)=\sqrt{[\int_0^t f(\tau)\sin(\omega\tau)d\tau]^2+[\int_0^t f(\tau)\cos(\omega\tau)d\tau]^2} \quad (1)$$

where Φ(t,ω) is the current amplitude- frequency spectrum, *t* is current time, ω is cyclic frequency, τ is the variable of integration, and *A* is the amplitude of harmonic components (Kharkevich, 2009).

From formula (1) it follows that initial phases φ of sine components A sin(ωt + φ) are not explicitly considered in finding the current amplitude-frequency spectrum. This may create difficulties for subsequent analytical calculations based on generated spectra, for example, in calculating harmonic amplitudes with required precision.

Accordingly, this article is aimed at identifying and assessing the influence of initial phases φ of harmonic components of the current amplitude-frequency spectrum Φ(t,ω) and developing recommendations

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for accounting for this influence. During the research, it will be assumed that at frequency ω, the generated spectrum is mainly influenced by a harmonic component with similar frequency, i.e., component A sin(ωt + φ).

**Problem resolution and its substantiation**

To identify and assess the influence of initial phases $\varphi$ of harmonic components on the current amplitude-frequency range Φ(t,ω), let us consider the influence of initial phase $\varphi$ of harmonic A sin(ωt + φ) on the current amplitude-frequency spectrum at some arbitrary frequency ω. For this purpose, let us substitute in formula (1) harmonic *A* sin(ωτ + φ), and consider spectrum formation for this harmonic in time.

After substituting harmonic *A* sin(ωτ + φ) as test signal f = f(t) in formula (1), formula (1) takes the following form

$$\Phi(t,\omega,\varphi) = A\sqrt{[\int_0^t \sin(\omega\tau+\varphi)\times\sin(\omega\tau)\,d\tau]^2+[\int_0^t \sin(\omega\tau+\varphi)\times\cos(\omega\tau d\tau)]^2} \quad (2)$$

Transforming the trigonometric expressions included into (2), and taking integrals $I_1$ $I_2$ from the results of transformation (3) and (4), we get:

sin (ωτ + φ) x sin (ωτ)= (sin ωτ x cosφ + cos ωτ x sin φ) x sin ωτ = sin² ωτ x cosφ + ½ sin 2ωτ x sin φ (3)

sin (ωτ + φ) x cos (ωτ)= (sin ωτ x cosφ + cos ωτ x sin φ) x cos ωτ = ½ sin 2ωτ x cos φ + cos² ωτ x sinφ (4)

$$I_1 = \int_0^t[\sin^2\omega\tau\times\cos\varphi+\frac{1}{2}\sin 2\omega\tau\times\sin\varphi]d\tau =$$

$$=\cos\varphi\int_0^t\sin^2\omega\tau d\tau+\frac{1}{2}\sin\varphi\int_0^t\sin 2\omega\tau d\tau =$$

$$=\frac{\cos\varphi}{\omega}\int_0^t\sin^2\omega\tau d\omega\tau+\frac{\sin\varphi}{4\omega}\int_0^t\sin 2\omega\tau d2\omega\tau =$$

$$=\frac{\cos\varphi}{\omega}\left(\frac{\omega t}{2}-\frac{\sin 2\omega t}{4}\right)+\frac{\sin\varphi}{4\omega}(-\cos 2\omega t+1) =$$

$$=\frac{t}{2}\cos\varphi-\frac{1}{4\omega}[\sin(2\omega t+\varphi)-\sin\varphi] =$$

$$=\frac{t}{2}\cos\varphi-\frac{1}{2\omega}\cos(\omega t+\varphi)\sin\omega t \quad (5)$$

$$I_2 = \int_0^t(\frac{1}{2}\sin 2\omega\tau\times\cos\varphi+\cos^2\omega\tau\times\sin\varphi)d\tau =$$

$$=\frac{1}{2}\cos\varphi\int_0^t\sin 2\omega\tau d\tau+\sin\varphi\int_0^t\cos^2\omega\tau d\tau =$$

$$=\frac{\cos\varphi}{4\omega}\int_0^t\sin 2\omega\tau d2\omega\tau+\frac{\sin\varphi}{\omega}\int_0^t\cos^2\omega\tau d\omega\tau =$$

$$=\frac{\cos\varphi}{4\omega}(-\cos 2\omega\tau+1)+\frac{\sin\varphi}{2}\left(\frac{\omega t}{2}+\frac{\sin 2\omega t}{4}\right) =$$

$$=\frac{t}{2}\sin\varphi-\frac{1}{4\omega}[\cos(2\omega t+\varphi)-\cos\varphi] =$$

$$=\frac{t}{2}\sin\varphi+\frac{1}{2\omega}\sin(\omega t+\varphi)\sin\omega t \quad (6)$$

By squaring the obtained values of integrals (5) and (6), and summing them, we get:

$$I_1^2+I_2^2 = [\frac{t}{2}\cos\varphi-\frac{1}{2\omega}\cos(\omega t+\varphi)\sin\omega t]^2+[\frac{t}{2}\sin\varphi+\frac{1}{2\omega}\sin(\omega t+\varphi)\sin\omega t]^2 =$$

$$=\frac{t^2}{4}-\frac{t}{2\omega}\sin\omega t[\cos(\omega t+\varphi)\cos\varphi-\sin(\omega t+\varphi)\sin\varphi]+\frac{\sin^2\omega t}{4\omega^2} =$$

$$=\frac{t^2}{4}-\frac{t}{2\omega}\cos(\omega t+2\varphi)\sin\omega t+\frac{1-\cos 2\omega t}{8\omega^2} \quad (7)$$

Substituting (7) into (2), we obtain the sought formula, which reflects formation of the current amplitude-frequency spectrum in time

$$\Phi = \Phi(t,\omega,\varphi) =$$

$$= A\sqrt{\frac{t^2}{4}-\frac{t}{2\omega}\cos(\omega t+2\varphi)\sin\omega t+\frac{1-\cos 2\omega t}{8\omega^2}} =$$

$$=\frac{At}{2}\sqrt{1-\frac{2}{\omega t}\cos(\omega t+2\varphi)\sin\omega t+\frac{1-\cos 2\omega t}{2(\omega t)^2}} =$$

$$=\frac{At}{2}\sqrt{1-\frac{1}{\omega t}[\sin(2\omega t+2\varphi)-\sin 2\varphi]+\frac{1-\cos 2\omega t}{2(\omega t)^2}} =$$

$$=\frac{At}{2}\sqrt{1-\frac{1}{2\pi N}[\sin(2\omega t+2\varphi)-\sin 2\varphi]+\frac{1-\cos 2\omega t}{2(2\pi N)^2}} \quad (8)$$

where *N* is the number of periods in sinusoidal component Asin(ωτ + φ) in time interval [0, *t*].

Let us consider expression (8) as function $f = f(x) = k\sqrt{x}$, where x is the independent variable, *k* is some constant, while component $\frac{1}{2\pi N}[\sin(2\omega t+2\varphi)-\sin 2\varphi]$ is error Δx of independent variable x at point *x* = 1. Then, neglecting value $\frac{1-\cos 2\omega t}{2(2\pi N)^2}$ as a value of higher order of smallness, as compared to value $\Delta x = \frac{1}{2\pi N}[\sin(2\omega t+2\varphi)-\sin 2\varphi]$, we can write

$$\frac{\Delta\Phi}{\Phi} = \frac{\Delta f}{f} = k\frac{\Delta x}{2\sqrt{x}} : \left(k\sqrt{x}\right) = \Delta x/2x \quad (9)$$

At point =1 , expression (9) takes the following form

$$\frac{\Delta\Phi}{\Phi} = \frac{\Delta x}{2} = \frac{1}{4\pi N}[\sin(2\omega t+2\varphi)-\sin 2\varphi \quad (10)$$

Due to the fact that | sin (2ωt + 2φ) - $\sin 2\varphi$ | ≤ 2, from (10) we get

$$\frac{\Delta\Phi}{\Phi} \le \frac{1}{4\pi N}\times 2 = \frac{1}{2\pi N} \quad (11)$$

Making similar calculations for the values of amplitudes *A* with the use of formula (8), in considering expression (8) as function $f = f(x) = k\sqrt{x}$ at point *x* = 1, we also get

$$\frac{\Delta A}{A} = \frac{\Delta x}{2x} \le \frac{1}{2\pi N} \quad (12)$$

## RESULTS AND DISCUSSION

From (8) it follows that the initial phase φ, the value of which for sinusoidal component *A* sin(ωτ + ) is unknown, influences the formation of the amplitude-frequency spectrum. This influence is by way of component $\frac{1}{2\pi N}[\sin(2\omega t+2\varphi)-\sin 2\varphi]$, and is reduced to exposing formed spectrum $\Phi = \Phi(t,\omega,\varphi)$ to pulses with frequency 2ω, and offsetting it by value sin 2φ, which, due to multiplier $\frac{1}{2\pi N}$ decrease their values with time.

The presence of component $\frac{1}{2\pi N}[\sin(2\omega t + 2\varphi) - \sin 2\varphi]$, where phase value φ is unknown, affects subsequent analytical calculations based on formula (8), for example, hampers calculation of amplitudes *A* with required precision.

The influence of component $\frac{1}{2\pi N}[\sin(2\omega t + 2\varphi) - \sin 2\varphi]$ on the value of spectrum Φ= Φ(t,ω,φ), which is formed based on formula (8), may be reduced by increasing time interval [0, *t*], i.e., the time of forming the amplitude-frequency spectrum. With time *t* that corresponds to *N* periods of the sinusoidal component, the influence of this component on the value of the formed amplitude-frequency spectrum, according to (11), will not exceed value $\frac{\Delta\Phi}{\Phi} \le \frac{1}{2\pi N}$. This allows, through choosing time of spectral analysis *t*, to reduce the influence of initial phases on the result of calculation by formula (8) to the desired values. So, for instance, if for forming an amplitude-frequency spectrum at frequency *ω,* time *t* is allocated, which corresponds to *N* = 16 periods, the uncertainty introduced by the influence of the initial phase of the harmonic on the value of the formed spectrum at this frequency will not exceed $\frac{\Delta\Phi}{\Phi} \le 0.01$. Accordingly, if amplitudes *A* are to be calculated with the use of formula (8) with the relative error not exceeding $\delta \le 0.01$, then, in accordance with formula (12) for forming the spectrum, it is necessary to allocate time that corresponds to 16 periods.

## CONCLUSION

As a result of the research performed in this article, it has been shown that the initial phases of harmonic components, the value of which in the studied signals is unknown, influence the formed spectrum. This influence is evident in the presence of pulsations and the constant component, which are superimposed on the formed spectrum, and the values of which decrease with time. Therefore, by choosing the time of spectrum formation, the above mentioned influence can be made weaker than a predefined value. This allows to perform analytical calculations based on the formed spectra with the error not exceeding the initially specified error.

The obtained result has certain generality, since it is valid, under initial assumptions, for any point of the amplitude-frequency spectrum, regardless of frequency ω.

The influence of harmonic components with frequency other than ω on the formed spectrum is expected to be assessed in future studies.

## REFERENCES


Antipov S.A., Gnezdilov D.S., Kozmin D.A. and Stopkin V.M. 2014. Digital interpolation algorithms of assessing harmonic signal frequency. Comparative analysis. *Radio engineering*. 3 : 42-46.

Britenkov A.K. and Stepanov N.S. 2014. Analiz zvukovyh i vibracionnyh signalov s pomoshh'ju obobshhjonnogo spektralno-analiticheskogo metoda [Analysis of acoustic and vibration signals with the help of generalized spectral-analytic method]. *Mechanical engineering and engineering education*. 3 : 20-31.

Diaconitsa S.A., Sugachevsky I.R. and Galashina M.M. 2013. Razrabotka metoda identifikacii mnogoparametricheskoj sistemy [Development of a multi-parameter system identification method]. *Systems. Methods. Technologies*. 2 : 70-75.

Filatov, K.V. and Filatov A.K. 2010. Measurement of acoustic frequency characteristics. *The World of Measurement*. 6 : 54-58.

Jurovich I., Zelensky A.A., Lukin V.V., Royenko A.A. and Simeunovich M. 2014. Dostoinstva i prilozhenija robastnogo diskretnogo preobrazovanija Furye [Advantages and applications of the robust discrete Fourier transformation]. *Physical bases of instrument engineering*. 3(1) : 26-41.

Kharkevich A.A. 2009. Spectra and analysis. (5-th edition). Moscow: Publishing house "LIBROKOM", pp. 112.

Lyubomudrov A.A. 2011. Metod spektralnogo analiza akusticheskih signalov dlja ogranichennogo diapazona chastot [Method of acoustic signals spectral analysis for a limited range of frequencies]. *Natural and technical Sciences*. 1 : 238-239.

Lyubomudrov A.A. and Bashkov A.A. 2011. Uvelichenie tochnosti metoda spektralnogo analiza [Increasing the accuracy of the spectral analysis method]. Bulletin of the Peoples' Friendship University of Russia. *Series: mathematics, informatics, physics*. 1 : 118-120.

Muravyev S.A. 2009. Spektralnyj analiz akusticheskih signalov, registriruemyh mnogojelementnymi skvazhnymi zondami [Spectral analysis of acoustic signals recorded with a multi-element borehole probes]. *Automation, remote control engineering and communication in oil industry*. 9 : 6-8.

Rastegaev, I.A., Merson D.L., Rastegaeva I.I., Vinogradov A.Y. and Chugunov A.V. 2012. Sravnitelnyj spektralnyj analiz shumopodobnyh akusticheskih signalov pri monitoringe i diagnostike promyshlennogo oborudovanija. [Comparative spectral analysis of noise-like acoustic signals for monitoring and diagnostics of industrial equipment]. *Monitoring. Diagnostics*. 10 : 80-87.

Rybochkin A.F. and Yakovlev A.I. 2011. Informacionno-chastotnye polosy pri analize akusticheskih signalov pchelinyh semej [Information and frequency bands in the analysis of acoustic signal of bee families]. *Alternative energy and ecology*. 2 : 81-89.

Ponomarev V.A., Ponomarev O.V., Ponomarev A.V. and Ponomarev N.V. 2014. Obobshhenie algoritmov Gercelja i skolzjashhego parametricheskogo diskretnogo preobrazovanija Furye [Generalization of Herzl algorithms and the sliding parametric discrete Fourier transformation]. *Digital signal processing*. 1 : 3-1.